Algebra

Introduction: About how many days each year does the temperature in Oklahoma City drop below zero? Water freezes at 0ϒC. How would you write a temperature below zero? You can write 1ϒC above zero as +1ϒC and 3ϒC below zero as -3ϒC. At 6:00 p.m. the temperature was 8ϒC above zero. Four hours later the temperature dropped 10ϒ. What was the temperature at 10:00 p.m.? Moving down 8ϒwould take you to zero and then 2ϒ more would give you the temperature of 2ϒC below zero that is written -2ϒC. There are many times in real life when you can use positive and negative numbers. The next few pages will review working with integers and solving simple equations.

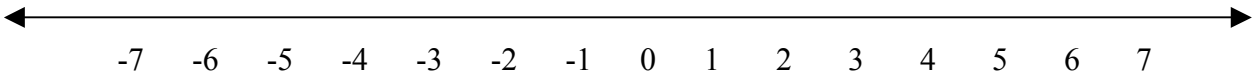
SECTION 1: THE MEANING AND USE OF SIGNED NUMBERS; THE SET OF INTEGERS

Part 1 Integers and the number line.

Every number has an opposite. On a number line any number and its opposite are the same distance from zero (0). The opposite of 5 is –5. The whole numbers and their opposites are called integers.

If you were to look at a number line, the greater of two numbers is farther to the right on a horizontal number line. On a vertical number line (like a thermometer), the greater number is above the lesser number.

Use the symbols < (less than) or > (greater than) to compare two numbers.
Look at the number line. Compare the numbers.



Compare –5 and 6. -5 is to the left of 6 or 6 is to the right of –5. Therefore, -5 < 6.
Compare 0 and –2. -2 is to the left of 0 or 0 is to the right of -2. Therefore, -2 < 0.
Compare 8 and 6. 6 is to the left of 8 or 8 is to the right of 6. Therefore, 6 < 8.
Compare –4 and –12. -4 is to the right of –12 or -12 is to the left of –4.
Therefore, -12 < -4.

Arrange these numbers in order from least to greatest:
-6 0 -3 7 10

Solution: Write them in order from left to right as they would appear on a number line.

-6 -3 0 7 10

Part 2 **Adding integers.**

1. The sum of any integer and 0 is that integer.
   Practice. **Add the following.**

   $2 + 0 = 2$ $23 + 0 = 23$ $-6 + 0 = -6$ $0 + 5 = 5$ $-22 + 0 = -22$

2. To add 2 positive numbers, disregard the positive signs and add the numbers. The sign on the answer will be positive. In other words, if the numbers have the same sign, add and keep the sign.
   Practice. **Add the following**.

   $^{+}7 + ^{+}8 = ^{+}15$ $^{+}10 + ^{+}4 = ^{+}14$ $^{+}50 + ^{+}22 = ^{+}72$

3. To add 2 negative numbers, disregard the negative signs and add the numbers. The sign on the answer will be negative. In other words, if the numbers have the same sign, add and keep the sign.
   Practice. **Add the following.**

   $^{-}7 + ^{-}8 = ^{-}15$ $^{-}10 + ^{-}4 = ^{-}14$ $^{-}50 + ^{-}22 = ^{-}72$

4. To add a positive and a negative number, disregard the signs and subtract the smaller number from the larger number. Determine the sign of the answer by taking the sign of the number that is the farthest from zero. This will be the sign of the answer.
   Practice. **Add the following.**

   $^{-}12 + ^{+}8 =$ subtract 8 from 12

   $$\begin{array}{r} 12 \\ \underline{-\ 8} \\ 4 \end{array}$$

   since $^{-}12$ is farther from 0 than 8, the sign of the answer will be the sign of 12 which is negative

   answer: -4

   Practice. **Add the following**.

   $^{-}22 + ^{+}38 =$ subtract 22 from 38

   $$\begin{array}{r} 38 \\ \underline{-\ 22} \\ 16 \end{array}$$

   since 38 is farther from 0 than $^{-}22$, the sign of the answer will be the sign of 38 which is positive

   answer: 16 (It is also understood that you do do not need to write a positive sign.)

5. Here are the rules to add integers.

Adding an integer and 0	The sum of any integer and 0 is that integer	3 + 0 = 3	20 + 0 = 20	0 + -9 = -9
Adding 2 positive integers	Add the numbers and answer is positive	6 + 9 = 15	21 + 13 = 34	210 + 32 = 242
Adding 2 negative integers	Add the numbers and the answer is negative	$^{-}8 + ^{-}5 = ^{-}13$	$^{-}50 + ^{-}22 = ^{-}72$	$^{-}12 + ^{-}14 = ^{-}26$
Adding a positive and a negative integer	Subtract the numbers and the sign of the answer will be the sign of the number farther from zero	$^{+}19 + ^{-}14 = ^{+}5$	$^{+}30 + ^{-}42 = ^{-}12$	$^{-}30 + ^{+}42 = ^{+}12$
Adding more than 2 integers	Add the positives. Add the negatives. Then find the sum.	$12 + 13 + ^{-}5 = 25 + ^{-}5 = 20$ $^{-}2 + 5 + ^{-}7 = ^{-}9 + 5 = ^{-}4$		

Part 3 Subtracting Integers.

Subtracting a number means adding its opposite. To subtract, follow these 2 rules:

1. Replace the number being subtracted by its opposite and change the subtraction symbol to addition.
2. Then add, remembering the rules for addition for integers. See part 2.

Practice. Subtract the following integers.

$7 - 3 = 7 + ^{-}3 = 4$ Replace 3, the number being subtracted , by its opposite, and change to addition. Follow rules for addition.

$^{-}8 - 10 = ^{-}8 + ^{-}10 = ^{-}18$ Replace 10, the number being subtracted, by its opposite, and change to addition. Follow rules for addition.

$8 - 10 = 8 + ^{-}10 = ^{-}2$ Replace 10, the number being subtracted, by its opposite, and change to addition. Follow rules for addition

$^{-}6 - \left(^{-}7\right) = ^{-}6 + \left(^{+}7\right) = 1$ Replace -7, the number being subtracted, by its opposite, and change to addition. Follow rules for addition

$^{-}5 - 3 - 10 = ^{-}5 + ^{-}3 + ^{-}10 = ^{-}18$ Replace 3 and 10 by its opposite, and change to addition. Follow the rules for addition.

Part 4 Multiplying Integers.

1. Multiplying by zero (0). The product of any number and zero (0) is zero (0).

   Practice. Multiply.

   $3 \cdot 0 = 0$ $4 \times 0 = 0$ $0 \times^{-} 5 = 0$

2. Multiplying two integers.
   Step 1: Multiply as if the numbers have no signs.
   Step 2: a. If both numbers have the same sign (both + or both -), then the product (answer) is positive (+).
   b. If both numbers have different signs (one + and one -), then the product (answer) is negative (-).

   Practice. Multiply.

   $3 \times 5 = 15$ $-3 \times -5 = 15$ $3 \times -5 = -15$ $-3 \times 5 = -15$

3. Multiplying more than two integers.
   Step 1: Multiply all the numbers disregarding the signs.
   Step 2: The answer is positive (+) if the number of negative factors is even.
   The answer is negative (-) if the number of negative factors is odd.

   Practice. Multiply.

   (-2)(-3)(4)(5) = 2x3=6 6x4=24 24x5=120 even # of negative factors = +120
   (-2)(1)(-4)(-2)= 2x1=2 2x4=8 8x2=16 odd # of negative factors = -16

Part 5 Dividing Integers.

1. Dividing by zero (0). Division by zero (0) is not possible. The answer is undefined. For example: $\frac{2}{0}$ = undefined.

2. Dividing two integers.
   Step 1: Divide as if the numbers have no signs.
   Step 2: a. If both numbers have the same sign (both + or both -), then the product (answer) is positive (+).
   c. If both numbers have different signs (one + and one -), then the product (answer) is negative (-).

   Practice. Divide.

   $20 \div -5 = -4$ $-12 \div -4 = 3$ $-15 \div 5 = -3$ $18 \div 3 = 6$

Summary of Part 4 and Part 5. **Multiplying and Dividing Integers.**

The following is a summary chart for multiplying or dividing integers.

$+ \cdot + = +$	$+ \div + = +$
$- \cdot - = +$	$- \div - = +$
$- \cdot + = -$	$- \div + = -$
$+ \cdot - = -$	$+ \div - = -$

Part 6 **Factors and Exponents**

When 2 or more numbers are multiplied, each of the numbers is called a factor. When a number can be expressed as a product of equal factors, except 1, the number is called a power. Here is an example:

First power:	$5^1 = 5$
Second power:	$5^2 = 5 \times 5 = 25$
Third power:	$5^3 = 5 \times 5 \times 5 = 125$
Fourth power:	$5^4 = 5 \times 5 \times 5 \times 5 = 625$

1. An *exponent* shows how many times a *base* is used as a factor.

exponent → 6^2 means 6×6 ← base

This is read "6 raised to the second power" or "6 squared".

exponent → 6^3 means $6 \times 6 \times 6$ ← base

This is read "6 raised to the third power" or "6 to the third power" or "**6 cubed**".

Practice. **Evaluate each expression.**

Expression	Arithmetic	Answer
3^4	$3 \times 3 \times 3 \times 3 =$	81
$4(3^2)$	$4 \times 3 \times 3 =$	36
$(-2)^3$	$(-2) \times (-2) \times (-2) =$	-8
-2^3	$(-1)(2^3)=(-1)(2)(2)(2) =$	-8
$5^2 \cdot 2^3$	$5 \times 5 \times 2 \times 2 \times 2 = 25 \times 8 =$	200
$(2 \cdot 4)^2$	$(8)^2 = 8 \times 8 =$	64
-2^2	$(-1)(2^2) = -1 \times 2 \times 2 =$	-4
$(-2)^2$	$(-2)(-2) =$	4

2. To evaluate an algebraic expression, replace the variable(s) by their numerical values. Then perform the operations that are shown.

Practice. Evaluate each expression when x = 3 and y = 4.

Expression	Arithmetic	Answer
$x(y^2)$	$3(4^2) = 3\ (\ 4 \times 4) = 3\ (16) =$	48
$(xy)^2$	$(3 \times 4)^2 = (12)^2 = 12 \times 12 =$	144
x^2y^2	$3^2 \times 4^2 = 3 \times 3 \times 4 \times 4 = 9 \times 16 =$	144
x^2y	$3^2 \times 4 = 3 \times 3 \times 4 = 9 \times 4 =$	36
$-x^3$	$(-1)3^3 = -1 \times 3 \times 3 \times 3 = -1 \times 27 =$	-27

Part 7 Order of Operations

Sometimes you will evaluate expressions involving grouping symbols and powers and other operations. If there is more than one operation (+ , - , x or ÷), the operations follow the order of operations which are as follows:

1. Do the operations inside parentheses first.
2. Do all exponents or powers.
3. Do the multiplication and/or divisions in order from left to right.
4. Do the additions and/or subtractions in order from left to right.

A favorite phrase that teachers like to remind students about the order of operations is "***P***lease ***E***xcuse ***M***y ***D***ear ***A***unt ***S***ally". Follow this order!!!

Parentheses

Exponents

Multiply or ***D***ivide from left to right

Add or ***S***ubtract from left to right

Practice. Simplify using order of operations.

1. $2 + 4 + 6 \div 2 - 1 =$
   $2 + 4 + 3 - 1 =$
   $6 + 3 - 1 =$
   $9 - 1 =$
   8

2. $2 + (4 + 6) \div 2 - 1 =$
   $2 + 10 \div 2 - 1 =$
   $2 + 5 - 1 =$
   $7 - 1 =$
   6

3. 3. $3 \cdot 2^2 - 5 \cdot 1 - 4 =$
   $3 \cdot 4 - 5 \cdot 1 - 4 =$
   $12 - 5 - 4 =$
   $7 - 4 =$
   3

4. $(6^2 - 14 + 3) \div (5^2 + 0) - 1 =$
   $(36 - 14 + 3) \div (25 + 0) - 1 =$
   $(22 + 3) \div (25) - 1 =$
   $(25) \div (25) - 1 =$
   $1 - 1 =$
   0

5. $3(2 + 3) - 5^2 + 4 \div 2 =$
   $3\ (5) - 5^2 + 4 \div 2 =$
   $3\ (5) - 25 + 4 \div 2 =$
   $15 - 25 + 2 =$
   $-10 + 2 =$
   -8

Section 2: Solving Equations

Introduction: There are many times when you will need to apply the strategies of solving problems to real-world situations. Problems may relate to sports, meteorology, profit and loss, physics, health careers, electricity, or graphic design, just to name a few. To solve problems, you will need to write an equation, solve it, and check to make sure that the answer makes sense. The next section will review solving equations.

An equation is a mathematical sentence stating that two quantities are equal. When an equation contains a variable, a value of the variable that makes the equation true is called the solution of the equation. When you find the solution of an equation, you are solving the equation for the answer.

To begin writing and solving equations, you will need to decide on which operation to use. The four basic operations are addition (+), subtraction (-), multiplication (x), and division (÷). Some of the key words that show which operations to use are in the chart below.

Addition	**Subtraction**	**Multiplication**	**Division**
Plus	minus	times	divide
Added to	subtracted from	multiplied by	divided by
Sum	difference	product	divided into
And	deduct	of	quotient
Altogether	remains	in all	each
Both	change	multiply	per
Combined	left	total	average
In all	lost	twice	cut
Increased	more than	percents	equal pieces
Total	nearer(…er)	whole	split

Another word that you will need to know has several different meanings is the word ***EQUAL.*** Some words that mean equal are: is, are, equal to, equivalent, same as, and the symbol =.

Part 1 Writing Algebraic Expressions

1. You will use symbols to translate word phrases into algebraic expressions.
2. Since it could be confusing to write 2 x *x*, there are other ways to write "2 times the number *x*": 2· x or 2(x) or (2)(x) or 2x. The most commonly used is 2x.
3. Parentheses ( ) are used to group numbers.

Practice **Writing Algebraic Expressions**

	Word Phrase	Answer
1.	the sum of x and 2	x + 2
2.	the difference of n and 4	n – 4
3.	the product of y and 3	3y or 3(y)
4.	the quotient of b and 7	$\frac{b}{7}$ or $b \div 7$
5.	a number increased by 7	n + 7
6.	4 more than a number	n + 4
7.	a number decreased by 2	n – 2
8.	5 less than twice a number	2n – 5
9.	the difference of 4 and n, increased by 3	(4 – n) + 3
10.	6 times the sum of x and 4	6(x + 4)
11.	twice the difference of y and 3	2(y – 3)
12.	15 more than a number, multiplied by 4	4(n + 15)

Note: The expression "a number" can be represented by any variable.

Part 2 **Solving Equations by Doing the Inverse by working Backwards**

When solving equations, the main goal is to get the variable on one side of the equation by itself. To begin to solve an equation, think backwards. Do the inverse of what was done to the variable. Follow all rules for integers.

1. Solve. x + 6 = 13

What was done to variable. 1. Added 6

Do Inverse. 1. subtract 6

x + 6 = 13
- 6 - 6

x = 7

2. Solve. x – 5 = 2

1. Subtracted 5

1. add 5

x – 5 = 2
+5 +5

x = 7

If there is more than one step or operations, think what was done to the variable. Write these down. Then to solve, work backwards by doing the inverse of each.

3. Solve: 3x – 5 = 10

1. multiplied by 3
2. subtract 5

1. add 5
2. divide by 3

3x – 5 = 10
+ 5 +5

3x = 15 → $\frac{3x}{3} = \frac{15}{3}$ → x = 5

What was done to variable. **Do Inverse.**

4. Solve: $9x = 72$ 1. multiplied by 9 1. divide by 9

$9x = 72$

$\frac{9x}{9} = \frac{72}{9}$ → $x = 8$

5. Solve: $\frac{x}{7} = -5$ 1. divided by 7 1. multiply by 7

$\frac{x}{7} = -5$ → $7 \cdot \frac{x}{7} = 7 \cdot (-5)$ → $x = -35$

6. Solve: $3(x - 2) = 24$ 1. subtract 2 1. divide by 3
2. multiply by 3 2. add 2

$3(x - 2) = 24$ → $\frac{3(x-2)}{3} = \frac{24}{3}$ → $x - 2 = 8$ →

$x - 2 = 8$

$+2 \quad +2$

$x = 10$

7. Solve: $\frac{x}{4} - 5 = 20$ 1. divided by 4 1. add 5

2. subtract 5 2. multiply by 4

$\frac{x}{4} - 5 = 20$ → $\frac{x}{4} - 5 = 20$

$+5 \quad +5$

$\frac{x}{4} = 25$ → $4(\frac{x}{4}) = 4(25)$ → $x = 100$

8. Solve: $\frac{x}{7} + 12 = 14$ 1. divide by 7 1. subtract 12

2. add 12 2. multiply by 7

$\frac{x}{7} + 12 = 14$

$-12 \quad -12$

→ $\frac{x}{7} = 2$ → $7(\frac{x}{7}) = 7(2)$

$x = 14$

Algebra Section 1

Practice Part 1

Use > , < , or = to compare the numbers.

1. -12 ____ -15

2. -22 ____ 0

3. 0 ____ -5

4. -6 ____ -6

5. 12 ____ -2

Arrange the numbers in order from least to greatest.

6. -5 0 7 -12 4

7. -22 -32 -1 -5 -20

8. 0 1 -2 5 10

Answers Part 1

1. >

2. <

3. >

4. =

5. >

6. -12 -5 0 4 7

7. -32 -22 -20 -5 -1

8. -2 0 1 5 10

Practice Part 2

1. 8 + 7
2. -8 + -7
3. -10 + 15
4. 12 + -14
5. 0 + 3
6. –150 + ¯140
7. ¯22 + ¯13 + ¯15
8. 50 + 20 + ¯14
9. –22 + 38
10. 12 + 13 + ¯5 + ¯4

Answers Part 2

1. 15
2. -15
3. 5
4. –2
5. 3
6. –290
7. –50
8. 56
9. 16
10. 16

Practice Part 3

1. 8 - 5
2. -8 - 12
3. –8 - ¯12
4. 9 - 5
5. –5 - ¯3
6. 9 - ¯5
7. 5 - ¯3
8. 0 - 2
9. 15 - 19
10. 20 - 30 - 42

Answers Part 2

1. 3
2. -8 + ¯12 = -20
3. -8 + 12 = 4
4. 4
5. –5 + 3 = -2
6. 9 + 5 = 14
7. 5 + 3 = 8
8. 0 + ¯2 = ¯2
9. 15 + ¯19 = ¯4
10. 20 + ¯30 + ¯42 = 20 + ¯72 = -52

Practice Part 4 and Part 5

1. 3 x 6
2. 3 x –6
3. –12 x –10
4. –8 x 2
5. 5 x 0
6. 20 ÷ -5
7. –20 ÷ -5
8. –100 ÷ 4
9. 15 ÷ 5
10. 8 ÷ 0

Answers Part 4 and Part 5

1. 18
2. -18
3. 120
4. –16
5. 0
6. –4
7. 4
8. –25
9. 3
10. undefined (no answer)

Practice Part 6

1. 3^3
2. $4(2^2)$
3. -3^2
4. $5^2 \cdot 2^3$
5. $-2^2 \cdot 3$

Evaluate for x = 2 and y = 3.

6. xy^2
7. x^2y
8. x^2y^2
9. x^3y
10. $-x^2y^3$

Answers Part 6

1. 3 x 3 x 3 = 27
2. 4 x 2 x 2 = 16
3. –1 x 3 x 3 = -9
4. 5 x 5 x 2 x 2 x 2 = 25 x 8 = 200
5. –1 x 2 x 2 x 3 = -12
6. 2 x 3 x 3 =18
7. 2 x 2 x 3 = 12
8. 2 x 2 x 3 x 3 = 36
9. 2 x 2 x 2 x 3 = 24
10. –1 x 2 x 2 x 3 x 3 x 3 = -108

Practice Part 7

1. $2 + 5 - 3 + 7 - 8$

2. $2 + (5 - 3) \div 2 + 4 - 3$

3. $3 (2^2) - 5(2) + 4 - 5$

4. $2 (4 + 5) + 3^2 - 4 \cdot 2$

5. $3 (4 - 1) - 3^2 + 8 \div 2 + 1$

6. $9 \div 3 \times 2 + 5$

Answers Part 7

1. $2 + 5 - 3 + 7 - 8$
   $7 - 3 + 7 - 8$
   $4 + 7 - 8$
   $11 - 8$
   3

2. $2 + (5 - 3) \div 2 + 4 - 3$
   $2 + 2 \div 2 + 4 - 3$
   $2 + 1 + 4 - 3$
   $3 + 4 - 3$
   $7 - 3$
   4

3. $3 (2^2) - 5(2) + 4 - 5$
   $3(4) - 5(2) + 4 - 5$
   $12 - 10 + 4 - 5$
   $2 + 4 - 5$
   $6 - 5$
   1

4. $2(4 + 5) + 3^2 - 4 \cdot 2$
   $2(9) + 3^2 - 4 \cdot 2$
   $2(9) + 9 - 4 \cdot 2$
   $18 + 9 - 8$
   $27 - 8$
   19

5. $3 (4 - 1) - 3^2 + 8 \div 2 + 1$
   $3(3) - 3^2 + 8 \div 2 + 1$
   $3(3) - 9 + 8 \div 2 + 1$
   $9 - 9 + 4 + 1$
   5

6. $9 \div 3 \times 2 + 5$
   $3 \times 2 + 5$
   $6 + 5$
   11

Algebra Section 2

Practice Part 1

Write each word phrase as an algebraic expression.

1. the difference of y and 2
2. the product of x and –7
3. a number increased by 8
4. 4 less than 3 times a number
5. 7 times the sum of x and 4
6. the sum of a number and 12 is 18

Write 3 key words for each.

7. addition
8. subtraction
9. multiplication
10. division

Answers Part 1

1. $y - 2$
2. $-7x$
3. $n + 8$
4. $3x - 4$
5. $7(x + 4)$
6. $n + 12 = 18$

7 – 10. Refer to page 7 for answers.

Practice Part 2

Solve each equation.

1. $x + 10 = 12$

2. $x + 10 = 8$

3. $4x = 12$

4. $-4x = -20$

5. $\frac{x}{5} = 3$

6. $\frac{x}{-4} = -5$

7. $3x - 5 = 10$

8. $2(x + 4) = -12$

9. $\frac{x}{2} - 4 = 13$

10. $\frac{x}{7} + 12 = 14$

Answers Part 2

1. $x = 2$

2. $x = -2$

3. $x = 3$

4. $x = 5$

5. $x = 15$

6. $x = 20$

7. $x = 5$

8. $x = -10$

9. $x = 34$

10. $x = 14$